Exercise 3.1

Q1 Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for each of these functions:

a) $y = x^3$ b) $y = x^5$ c) $y = x^4$ d) $y = x$

e) $y = \frac{1}{x}$ f) $y = \sqrt{x}$ g) $y = \frac{1}{x^2}$ h) $y = x\sqrt{x}$

Q2 Find $f'(x)$ and $f''(x)$ for each of these functions:

a) $f(x) = x(4x^2 - x)$ b) $f(x) = (x^2 - 3)(x - 4)$

c) $f(x) = \frac{4x^5 + 12x^3 - 40x}{4(x^2 + 5)}$ d) $f(x) = 3\sqrt{x} + x\sqrt{x}$

e) $f(x) = \frac{1}{x}(3x^4 - 2x^3)$ f) $f(x) = \frac{x^2 - x\sqrt{x} + 7x}{\sqrt{x}}$

Q3 Find the value of the second derivative at the given value for x.

a) $f(x) = x^3 - x^2,\ x = 3$ b) $y = x\sqrt{x} - \frac{1}{x},\ x = 4$

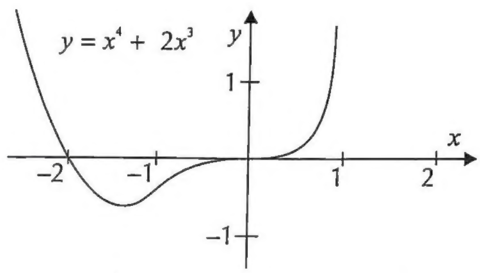
c) $f(x) = x^2(x - 5)(x^2 + x),\ x = -1$ d) $y = \frac{x^5 + 4x^4 - 12x^3}{x + 6},\ x = 5$

e) $f(x) = \frac{9x^2 + 3x}{3\sqrt{x}},\ x = 1$ f) $y = \left(\frac{1}{x^2} + \frac{1}{x}\right)(5 - x),\ x = -3$

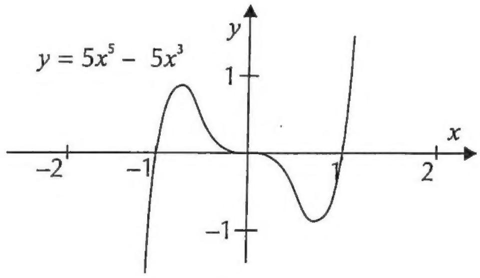
Exercise 3.2

Q1 Without doing any calculations, say how many stationary points the graphs below have in the intervals shown.

a)



b)



Q2 Find the x-coordinates of the stationary points of the curves with the following equations:

a) $y = x^2 + 3x + 2$ b) $y = (3 - x)(4 + 2x)$

Q3 Find the coordinates of the stationary points of the curves with the following equations:

a) $y = 2x^2 - 5x + 2$ b) $y = -x^2 + 3x - 4$

c) $y = 7 - 6x - 3x^2$ d) $y = (x - 1)(2x + 3)$

Q4 Find the coordinates of the stationary points of the curves with the following equations:

a) $y = x^3 - 3x + 2$ b) $y = 4x^3 + 5$

Q5 Hint: If there are no stationary points, there are no values of x for which $f'(x) = 0$.

Q5 Show that the graph of the function given by $f(x) = x^5 + 3x + 2$ has no stationary points.

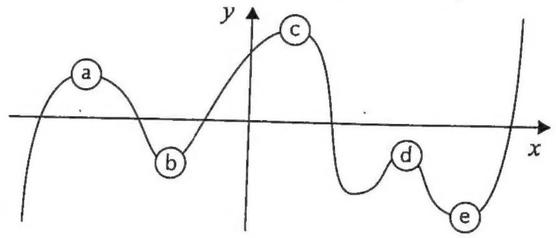
Q6 a) Differentiate $y = x^3 - 7x^2 - 5x + 2$.

b) Hence find the coordinates of the stationary points of the curve with equation $y = x^3 - 7x^2 - 5x + 2$.

Q7 A graph is given by the function $f(x) = x^3 + kx$, where k is a constant. Given that the graph has no stationary points, find the range of possible values for k. PROBLEM SOLVING

Exercise 3.3

Q1 The diagram below shows a sketch of the graph of $y = f(x)$. For each turning point, say whether $f''(x)$ would be positive or negative.



Q2 For each of the following, find the second derivative of the function and say whether the given point is a maximum or a minimum:

a) $y = x^3 - 12x + 4$ has a stationary point at (2, –12).

b) $y = 2x^4 - 16x^3 + 900$ has a stationary point at (6, 36).

c) $y = 4x^5 + 15x^4 - 250$ has a stationary point at (–3, –7).

d) $y = x^5 - 5x^4 + 5x^2 - 40x + 400$ has a stationary point at (4, 64).

Q3 A function $y = f(x)$ is such that $f(1) = 3$, $f'(1) = 0$ and $f''(1) = 7$.

a) Give the coordinates of one of the turning points of $f(x)$.

b) Determine the nature of this turning point, explaining your answer.

Q4 Find the stationary points on the graphs of the following functions and say whether they're maximum or minimum turning points:

a) $y = 5 - x^2$

b) $y = 2x^3 - 6x + 2$

c) $y = x^3 - 3x^2 - 24x + 15$

d) $y = x^4 + 4x^3 + 4x^2 - 10$

Q5 Find the stationary points on the graphs of the following functions and say whether they're maximum or minimum turning points:

a) $f(x) = 8x^3 + 16x^2 + 8x + 1$

b) $f(x) = \frac{27}{x^3} + x$

Q6 a) Given that $f(x) = x^3 - 3x^2 + 4$, find $f'(x)$ and $f''(x)$.

b) Hence find the coordinates of any stationary points on the graph $f(x)$ and say whether they're maximum or minimum turning points.

Q7 A function is given by $y = x^2 + \frac{2000}{x}$.

a) Find the value of x at which y is stationary.

b) Is this a minimum or maximum point?

Q8 The curve given by $f(x) = x^3 + ax^2 + bx + c$ has a stationary point with coordinates (3, 10). If $f''(x) = 0$ at (3, 10), find a, b and c. PROBLEM SOLVING

Q9 a) Given that a curve with the equation $y = x^4 + kx^3 + x^2 + 17$ has only one stationary point, show that $k^2 < \frac{32}{9}$. PROBLEM SOLVING

b) Find the coordinates of the stationary point and say whether it's a maximum or a minimum point.

3. Using Differentiation

Exercise 3.1 — Finding second order derivatives

Q1 a) $\frac{dy}{dx} = 3x^2$ and $\frac{d^2y}{dx^2} = 6x$.

b) $\frac{dy}{dx} = 5x^4$ and $\frac{d^2y}{dx^2} = 20x^3$.

c) $\frac{dy}{dx} = 4x^3$ and $\frac{d^2y}{dx^2} = 12x^2$.

d) $\frac{dy}{dx} = 1$ and $\frac{d^2y}{dx^2} = 0$.

e) $y = x^{-1}$, so $\frac{dy}{dx} = -x^{-2} = -\frac{1}{x^2}$ and $\frac{d^2y}{dx^2} = 2x^{-3} = \frac{2}{x^3}$.

f) $y = x^{\frac{1}{2}}$, so $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$
and $\frac{d^2y}{dx^2} = -\frac{1}{4}x^{-\frac{3}{2}} = -\frac{1}{4(\sqrt{x})^3}$.

g) $y = x^{-2}$, so $\frac{dy}{dx} = -2x^{-3} = -\frac{2}{x^3}$ and $\frac{d^2y}{dx^2} = 6x^{-4} = \frac{6}{x^4}$.

h) $y = x\sqrt{x} = x^1x^{\frac{1}{2}} = x^{1+\frac{1}{2}} = x^{\frac{3}{2}}$,
so $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$ and $\frac{d^2y}{dx^2} = \frac{3}{4}x^{-\frac{1}{2}} = \frac{3}{4\sqrt{x}}$.

Q2 a) $f(x) = x(4x^2 - x) = 4x^3 - x^2$
$f'(x) = 12x^2 - 2x$
$f''(x) = 24x - 2$

b) $f(x) = (x^2 - 3)(x - 4) = x^3 - 4x^2 - 3x + 12$
$f'(x) = 3x^2 - 8x - 3$
$f''(x) = 6x - 8$

c) $f(x) = \frac{4x^5 + 12x^3 - 40x}{4(x^2+5)} = \frac{4x(x^4 + 3x^2 - 10)}{4(x^2+5)}$
$= \frac{4x(x^2+5)(x^2-2)}{4(x^2+5)} = x(x^2-2) = x^3 - 2x$
$f'(x) = 3x^2 - 2$
$f''(x) = 6x$

d) $f(x) = 3x^{\frac{1}{2}} + xx^{\frac{1}{2}} = 3x^{\frac{1}{2}} + x^{\frac{3}{2}}$
$f'(x) = \frac{3}{2}x^{-\frac{1}{2}} + \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2\sqrt{x}} + \frac{3}{2}\sqrt{x}$
$f''(x) = \frac{3}{2}(-\frac{1}{2}x^{-\frac{3}{2}}) + \frac{3}{2}(\frac{1}{2}x^{-\frac{1}{2}}) = -\frac{3}{4}x^{-\frac{3}{2}} + \frac{3}{4}x^{-\frac{1}{2}}$
$= -\frac{3}{4(\sqrt{x})^3} + \frac{3}{4\sqrt{x}} \left(= -\frac{3}{4x\sqrt{x}} + \frac{3}{4\sqrt{x}}\right)$

e) $f(x) = \frac{1}{x}(3x^4 - 2x^3) = 3x^3 - 2x^2$
$f'(x) = 9x^2 - 4x$
$f''(x) = 18x - 4$

f) $f(x) = \frac{x^2 - xx^{\frac{1}{2}} + 7x}{x^{\frac{1}{2}}} = x^2x^{-\frac{1}{2}} - xx^{\frac{1}{2}}x^{-\frac{1}{2}} + 7xx^{-\frac{1}{2}}$
$= x^{\frac{3}{2}} - x + 7x^{\frac{1}{2}}$
$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - 1 + 7(\frac{1}{2}x^{-\frac{1}{2}}) = \frac{3}{2}\sqrt{x} - 1 + \frac{7}{2\sqrt{x}}$
$f''(x) = \frac{3}{2}(\frac{1}{2}x^{-\frac{1}{2}}) + \frac{7}{2}(-\frac{1}{2}x^{-\frac{3}{2}}) = \frac{3}{4\sqrt{x}} - \frac{7}{4(\sqrt{x})^3}$

Q3 a) $f'(x) = 3x^2 - 2x$, so $f''(x) = 6x - 2$.
$f''(3) = 16$.

b) $y = xx^{\frac{1}{2}} - x^{-1} = x^{\frac{3}{2}} - x^{-1}$ so $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + x^{-2}$
so $\frac{d^2y}{dx^2} = \frac{3}{2}(\frac{1}{2}x^{-\frac{1}{2}}) - 2x^{-3} = \frac{3}{4\sqrt{x}} - \frac{2}{x^3}$
so at $x = 4$, $\frac{d^2y}{dx^2} = \frac{11}{32}$.

c) $f(x) = x^2(x^3 - 4x^2 - 5x) = x^5 - 4x^4 - 5x^3$
so $f'(x) = 5x^4 - 16x^3 - 15x^2$
and $f''(x) = 20x^3 - 48x^2 - 30x$.
$f''(-1) = -38$.

d) $y = \frac{x^3(x+6)(x-2)}{(x+6)} = x^3(x-2) = x^4 - 2x^3$
so $\frac{dy}{dx} = 4x^3 - 6x^2$, $\frac{d^2y}{dx^2} = 12x^2 - 12x$.
At $x = 5$, $\frac{d^2y}{dx^2} = 240$.

e) $f(x) = \frac{9x^2 + 3x}{3\sqrt{x}} = 3x^{\frac{3}{2}} + x^{\frac{1}{2}}$ so
$f'(x) = 3(\frac{3}{2}x^{\frac{1}{2}}) + \frac{1}{2}x^{-\frac{1}{2}} = \frac{9}{2}\sqrt{x} + \frac{1}{2\sqrt{x}}$ and so
$f''(x) = \frac{9}{2}(\frac{1}{2}x^{-\frac{1}{2}}) + \frac{1}{2}(-\frac{1}{2}x^{-\frac{3}{2}}) = \frac{9}{4\sqrt{x}} - \frac{1}{4(\sqrt{x})^3}$
$f''(1) = 2$.

f) $y = (x^{-2} + x^{-1})(5 - x)$
$= 5x^{-2} - x^{-2}x + 5x^{-1} - xx^{-1}$
$= 5x^{-2} - x^{-1} + 5x^{-1} - 1 = 5x^{-2} + 4x^{-1} - 1$
$\frac{dy}{dx} = 5(-2x^{-3}) + 4(-x^{-2}) = -10x^{-3} - 4x^{-2}$
so $\frac{d^2y}{dx^2} = 30x^{-4} + 8x^{-3} = \frac{30}{x^4} + \frac{8}{x^3}$.
At $x = -3$, $\frac{d^2y}{dx^2} = \frac{2}{27}$.

Exercise 3.2 — Stationary points

Q1 a) The graph has 2 stationary points — a minimum anc a point of inflection.

b) The graph has 3 stationary points — a maximum, a minimum and a point of inflection.

Q2 a) $\frac{dy}{dx} = 2x + 3$. When $\frac{dy}{dx} = 0$, $2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

b) $y = (3 - x)(4 + 2x) = 12 + 2x - 2x^2$
$\frac{dy}{dx} = 2 - 4x$. When $\frac{dy}{dx} = 0$, $2 - 4x = 0 \Rightarrow x = \frac{1}{2}$

Q3 a) $\frac{dy}{dx} = 4x - 5$. When $\frac{dy}{dx} = 0$, $4x - 5 = 0 \Rightarrow x = \frac{5}{4}$
When $x = \frac{5}{4}$, $y = 2(\frac{5}{4})^2 - 5(\frac{5}{4}) + 2 = -\frac{9}{8}$
So the coordinates are $(\frac{5}{4}, -\frac{9}{8})$.

b) $\frac{dy}{dx} = -2x + 3$. When $\frac{dy}{dx} = 0$, $-2x + 3 = 0$
$\Rightarrow x = \frac{3}{2}$. When $x = \frac{3}{2}$, $y = -(\frac{3}{2})^2 + 3(\frac{3}{2}) - 4 = -\frac{7}{4}$
So the coordinates are $(\frac{3}{2}, -\frac{7}{4})$.

c) $\frac{dy}{dx} = -6 - 6x$. When $\frac{dy}{dx} = 0$, $-6 - 6x = 0 \Rightarrow x = -1$
When $x = -1$, $y = 7 - 6(-1) - 3(-1)^2 = 10$.
So the coordinates are $(-1, 10)$.

d) $y = (x - 1)(2x + 3) = 2x^2 + x - 3$
$\frac{dy}{dx} = 4x + 1$. When $\frac{dy}{dx} = 0$, $4x + 1 = 0 \Rightarrow x = -\frac{1}{4}$
When $x = -\frac{1}{4}$, $y = (-\frac{1}{4} - 1)(2(-\frac{1}{4}) + 3) = -\frac{25}{8}$.
So the coordinates are $(-\frac{1}{4}, -\frac{25}{8})$.

Q4 a) $\frac{dy}{dx} = 3x^2 - 3$. When $\frac{dy}{dx} = 0$, $3x^2 - 3 = 0 \Rightarrow x = \pm 1$
When $x = 1$, $y = 1^3 - 3(1) + 2 = 0$.
When $x = -1$, $y = (-1)^3 - 3(-1) + 2 = 4$.
So the coordinates are $(1, 0)$ and $(-1, 4)$.

b) $\frac{dy}{dx} = 12x^2$. When $\frac{dy}{dx} = 0$, $12x^2 = 0 \Rightarrow x = 0$
When $x = 0$, $y = 4(0)^3 + 5 = 5$.
So the coordinates are $(0, 5)$.

Q5 $f'(x) = 5x^4 + 3$. When $f'(x) = 0$, $5x^4 + 3 = 0 \Rightarrow x^4 = -\frac{3}{5}$
Finding a solution would involve finding the fourth root of a negative number. But $x^4 = (x^2)^2$, so x^4 is alwa positive and so there are no stationary points.

Q6 a) $\frac{dy}{dx} = 3x^2 - 14x - 5$

b) When $\frac{dy}{dx} = 0$, $3x^2 - 14x - 5 = 0$
$\Rightarrow (3x + 1)(x - 5) = 0$, so $x = -\frac{1}{3}$ and $x = 5$.
When $x = -\frac{1}{3}$, $y = (-\frac{1}{3})^3 - 7(-\frac{1}{3})^2 - 5(-\frac{1}{3}) + 2 =$
When $x = 5$, $y = 5^3 - 7(5)^2 - 5(5) + 2 = -73$.
So the coordinates are $(-\frac{1}{3}, \frac{77}{27})$ and $(5, -73)$.

Q7 For stationary points to occur, $f'(x)$ must equal zero, s
$f'(x) = 3x^2 + k = 0 \Rightarrow -\frac{k}{3} = x^2$. For this equation to have a solution, k can't be positive (or it would be taking the square root of a negative number), so $k \leq$ C
Therefore, if the graph has no stationary points, $k > 0$

Exercise 3.3 — Maximum and minimum points

Q1 a) negative b) positive c) negative
d) negative e) positive

Q2 a) $\frac{dy}{dx} = 3x^2 - 12$ $\frac{d^2y}{dx^2} = 6x$
At $(2, -12)$, $\frac{d^2y}{dx^2} = 6 \times 2 = 12 > 0$,
so $(2, -12)$ is a minimum.

b) $\frac{dy}{dx} = 8x^3 - 48x^2$ $\frac{d^2y}{dx^2} = 24x^2 - 96x$
At $(6, 36)$, $\frac{d^2y}{dx^2} = 24 \times 6^2 - 96 \times 6 = 288 > 0$,
so $(6, 36)$ is a minimum.

c) $\frac{dy}{dx} = 20x^4 + 60x^3$ $\frac{d^2y}{dx^2} = 80x^3 + 180x^2$
At $(-3, -7)$, $\frac{d^2y}{dx^2} = 80 \times (-3)^3 + 180 \times (-3)^2$
$= -540 < 0$, so $(-3, -7)$ is a maximum.

d) $\frac{dy}{dx} = 5x^4 - 20x^3 + 10x - 40$
$\frac{d^2y}{dx^2} = 20x^3 - 60x^2 + 10$
At $(4, 64)$, $\frac{d^2y}{dx^2} = 20 \times 4^3 - 60 \times 4^2 + 10 = 330 > 0$,
so $(4, 64)$ is a minimum.

Q3 a) $(1, 3)$
All the clues are in the question — the derivative when $x = 1$ is zero so you know it's a stationary point, and the y-value when $x = 1$ is 3.

b) The second derivative at $x = 1$ is positive, so it's a minimum.

Q4 a) $\frac{dy}{dx} = -2x$. When $\frac{dy}{dx} = 0$, $x = 0$. When $x = 0$, $y = 5 - 0 = 5$. So the coordinates are $(0, 5)$.
$\frac{d^2y}{dx^2} = -2$, so it's a maximum turning point.

b) $\frac{dy}{dx} = 6x^2 - 6$. When $\frac{dy}{dx} = 0$, $6x^2 = 6 \Rightarrow x = \pm 1$
When $x = 1$, $y = 2 - 6 + 2 = -2$. When $x = -1$, $y = -2 + 6 + 2 = 6$. So the coordinates are $(1, -2)$ and $(-1, 6)$. $\frac{d^2y}{dx^2} = 12x$.
At $(1, -2)$, $\frac{d^2y}{dx^2} = 12$, so it's a minimum.
At $(-1, 6)$, $\frac{d^2y}{dx^2} = -12$ so it's a maximum.

c) $\frac{dy}{dx} = 3x^2 - 6x - 24$. When $\frac{dy}{dx} = 0$, $x^2 - 2x - 8 = 0$
$\Rightarrow (x - 4)(x + 2) = 0 \Rightarrow x = 4$ and -2.
When $x = 4$, $y = 64 - 48 - 96 + 15 = -65$.
When $x = -2$, $y = -8 - 12 + 48 + 15 = 43$.
So the coordinates are $(4, -65)$ and $(-2, 43)$.
$\frac{d^2y}{dx^2} = 6x - 6$. At $(4, -65)$, $\frac{d^2y}{dx^2} = 24 - 6 = 18$,
so it's a minimum.
At $(-2, 43)$, $\frac{d^2y}{dx^2} = -12 - 6 = -18$, so it's a maximum.

d) $\frac{dy}{dx} = 4x^3 + 12x^2 + 8x$.
When $\frac{dy}{dx} = 0$, $x^3 + 3x^2 + 2x = 0$
$\Rightarrow x(x + 2)(x + 1) = 0$, so $x = 0$, -1 and -2.
When $x = 0$, $y = 0 + 0 + 0 - 10 = -10$.
When $x = -1$, $y = 1 - 4 + 4 - 10 = -9$.
When $x = -2$, $y = 16 - 32 + 16 - 10 = -10$.
So the stationary points are $(0, -10)$, $(-1, -9)$ and $(-2, -10)$.
$\frac{d^2y}{dx^2} = 12x^2 + 24x + 8$. At $(0, -10)$,
$\frac{d^2y}{dx^2} = 0 + 0 + 8 = 8$, so it's a minimum.
At $(-1, -9)$, $\frac{d^2y}{dx^2} = 12 - 24 + 8 = -4$,
so it's a maximum.
At $(-2, -10)$, $\frac{d^2y}{dx^2} = 48 - 48 + 8 = 8$,
so it's a minimum.

Q5 a) $f'(x) = 24x^2 + 32x + 8$.
When $f'(x) = 0$, $3x^2 + 4x + 1 = 0$
$\Rightarrow (3x + 1)(x + 1) = 0$, so $x = -1$ and $-\frac{1}{3}$.
When $x = -1$, $f(x) = -8 + 16 - 8 + 1 = 1$.
When $x = -\frac{1}{3}$, $f(x) = -\frac{8}{27} + \frac{16}{9} - \frac{8}{3} + 1 = -\frac{5}{27}$.
So the coordinates are $(-1, 1)$ and $(-\frac{1}{3}, -\frac{5}{27})$.
$f''(x) = 48x + 32$. At $(-1, 1)$ $f''(x) = -48 + 32 = -16$, so it's a maximum.
At $(-\frac{1}{3}, -\frac{5}{27})$, $f''(x) = -\frac{48}{3} + 32 = 16$,
so it's a minimum.

b) $f(x) = \frac{27}{x^3} + x = 27x^{-3} + x \Rightarrow f'(x) = -81x^{-4} + 1$.
When $f'(x) = 0$, $x^4 = 81 \Rightarrow x = \pm 3$.
When $x = 3$, $f(x) = \frac{27}{27} + 3 = 4$.
When $x = -3$, $f(x) = -\frac{27}{27} - 3 = -4$.
So the coordinates are $(3, 4)$ and $(-3, -4)$.
$f''(x) = 324x^{-5}$. At $(3, 4)$ $f''(x) = \frac{4}{3}$,
so it's a minimum.
At $(-3, -4)$ $f''(x) = -\frac{4}{3}$, so it's a maximum.

Q6 a) $f'(x) = 3x^2 - 6x$. $f''(x) = 6x - 6$.

b) When $f'(x) = 0$, $3x^2 - 6x = 0 \Rightarrow x(x - 2) = 0$,
so $x = 0$ and $x = 2$. When $x = 0$, $f(x) = 0 - 0 + 4 = 4$.
When $x = 2$, $f(x) = 8 - 12 + 4 = 0$.
So the coordinates are $(0, 4)$ and $(2, 0)$.
At $(0, 4)$ $f''(x) = 0 - 6 = -6$, so it's a maximum.
At $(2, 0)$ $f''(x) = 12 - 6 = 6$, so it's a minimum.

Q7 a) $y = x^2 + \frac{2000}{x} = x^2 + 2000x^{-1} \Rightarrow \frac{dy}{dx} = 2x - \frac{2000}{x^2}$
When $\frac{dy}{dx} = 0$, $2x = \frac{2000}{x^2} \Rightarrow x^3 = 1000 \Rightarrow x = 10$

b) $\frac{d^2y}{dx^2} = 2 + \frac{4000}{x^3}$. When $x = 10$,
$\frac{d^2y}{dx^2} = 2 + 4 = 6$, so it's a minimum.

Q8 $f(x) = x^3 + ax^2 + bx + c \Rightarrow f'(x) = 3x^2 + 2ax + b$.
$\Rightarrow f''(x) = 6x + 2a$. At the point $(3, 10)$:
$10 = 3^3 + a(3^2) + b(3) + c \Rightarrow 10 = 27 + 9a + 3b + c$
As $(3, 10)$ is a stationary point, $0 = 3(3^2) + 2a(3) + b$
$\Rightarrow 0 = 27 + 6a + b$. We know that $f''(3) = 0$,
so $0 = 6(3) + 2a \Rightarrow 0 = 18 + 2a \Rightarrow a = -9$.
Then $0 = 27 + 6a + b = 27 + 6(-9) + b \Rightarrow b = 27$
And $10 = 27 + 9a + 3b + c = 27 + 9(-9) + 3(27) + c$
$\Rightarrow c = -17$. So $f(x) = x^3 - 9x^2 + 27x - 17$.

Q9 a) $\frac{dy}{dx} = 4x^3 + 3kx^2 + 2x$.
Stationary points occur when $\frac{dy}{dx} = 0$,
so $4x^3 + 3kx^2 + 2x = 0 \Rightarrow x(4x^2 + 3kx + 2) = 0$
so $x = 0$ or $4x^2 + 3kx + 2 = 0$.
As you know the only stationary point occurs at $x = 0$, the part in brackets can't have any solutions. This gives you information about the discriminant of the quadratic equation:
$b^2 - 4ac < 0 \Rightarrow 9k^2 < 32 \Rightarrow k^2 < \frac{32}{9}$.

b) When $x = 0$, $y = 0 + 0 + 0 + 17 = 17$, so the coordinates are $(0, 17)$.
$\frac{d^2y}{dx^2} = 12x^2 + 6kx + 2$. When $x = 0$, $\frac{d^2y}{dx^2} = 2$,
so it's a minimum.